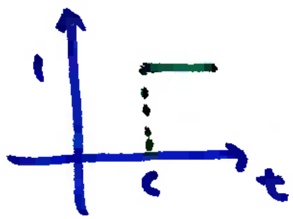
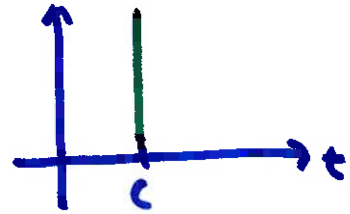
7.6 (continued)

$u_c(t)$



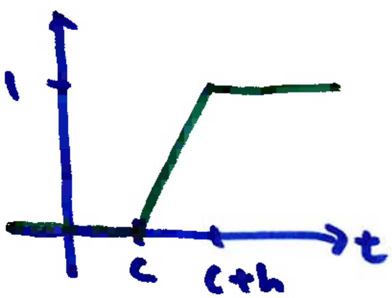
$\delta(t-c)$



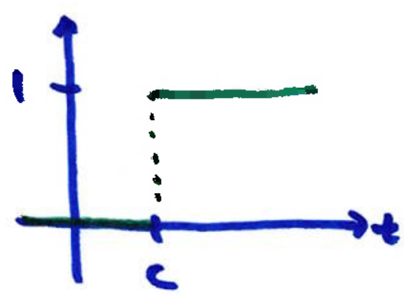
it turns out $\frac{d}{dt} u_c(t) = \delta(t-c)$

why?

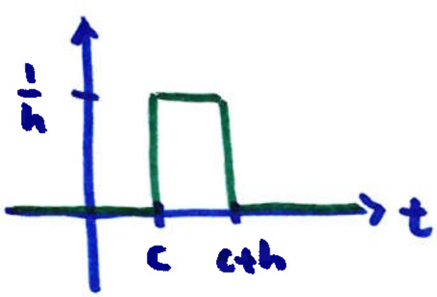
approx. $u_c(t)$ as a ramp



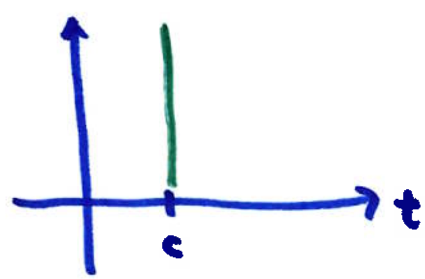
$\xrightarrow{h \to 0}$



deriv.



$\xrightarrow{h \to 0}$



what $f(t)$ has a Laplace transform of 1 ?

$$\mathcal{L}\{\delta(t-c)\} = e^{-cs}$$

if $c=0$, $\mathcal{L}\{\delta(t)\} = 1$ $\qquad$ $\delta(t)$ is impulse at $t=0$

mass-spring-damper $\quad ay''+by'+cy = \delta(t) \quad y(0)=y'(0)=0$

$$(as^2+bs+c)Y = 1$$

$$Y = \underbrace{\frac{1}{as^2+bs+c}}_{\text{impulse response}} \qquad y(t) = \mathcal{L}^{-1}\left\{\frac{1}{as^2+bs+c}\right\} = g(t)$$

$$ay''+by'+cy = f(t) \qquad y(0)=y'(0)=0$$

$$(as^2+bs+c)Y = F$$

$$Y = \underbrace{\frac{1}{as^2+bs+c}}_{\text{imp. response}} \cdot F$$

$$y(t) = \int_0^t g(t-\tau)\, f(\tau)\, d\tau$$

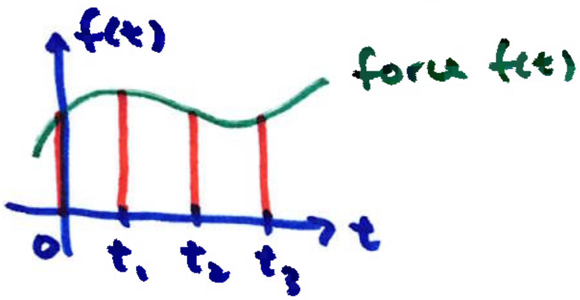
convolution

shifted impulse response

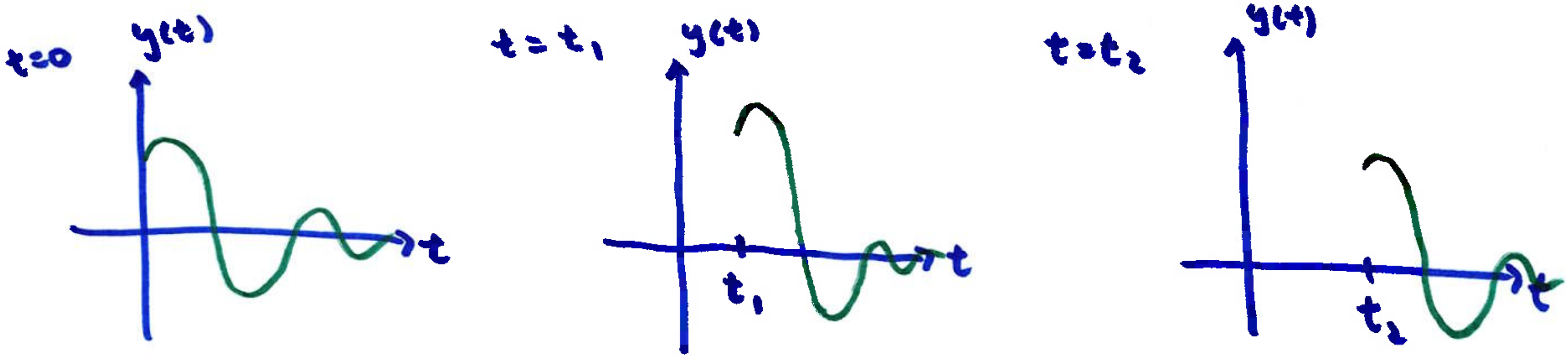
what is this doing?

sampling $f(t)$ as a bunch of impulses at different time

find impulse responses then stack them



$f(t)$: inf. many impulses under curve



convolution superimposes all onto one function of t

impulse response

$$y = \int_0^t g(t-\tau)\, f(\tau)\, d\tau$$

do some algebra in s domain and using properties of Laplace

deriv. of force

$$y = h(t)\, f(0) + \int_0^t h(t-\tau)\, f'(\tau)\, d\tau$$

step response

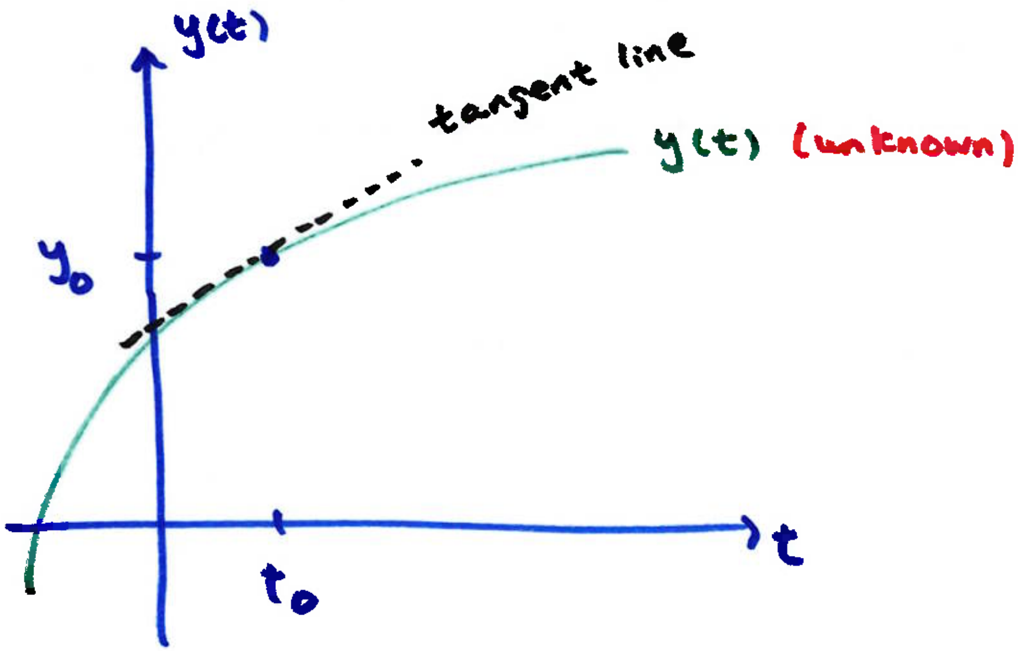
(response due to step input)

Duhamel's Principle

2.4 Euler's Method

numerical method to solve $y' = f(t, y)$ also applicable to higher-order

basic idea: tangent line approx.



we know its slope $y' = f(t, y)$ at (t, y)

suppose we know one point (initial condition) (t_0, y_0)

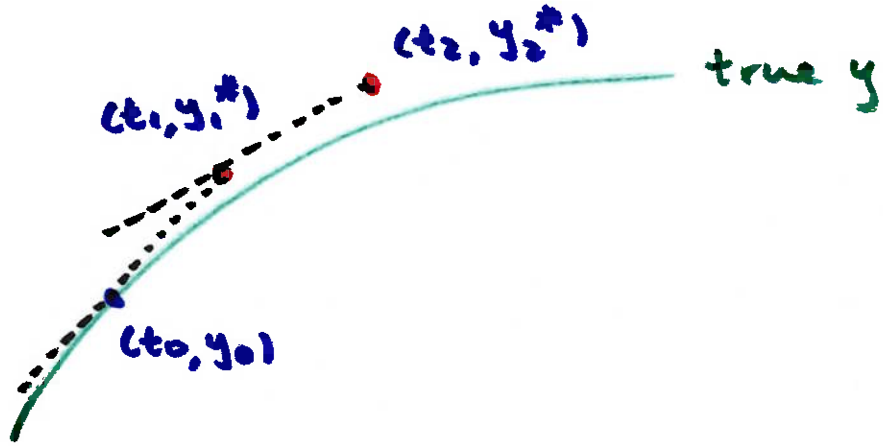
tangent line at (t_0, y_0) $\quad y - y_0 = f(t_0, y_0)(t - t_0)$

$$y = y_0 + f(t_0, y_0)(t - t_0)$$

$$\approx \text{true } y(t)$$

move a little bit on the tangent line $\rightarrow y(t_1) = y_1$

repeat as needed



y_1^* approx. of true y_1

typically we choose a fixed step size h : $t_n = t_{n-1} + h$

<u>example</u> $y' = 2y - 3t \quad y(0) = 1$

use $h = 0.25$ to estimate $y(0.5)$

$$\left.\begin{array}{l} t_0 = 0 \\ y_0 = 1 \end{array}\right\} \text{given}$$

$$t_1 = t_0 + h = 0 + 0.25 = 0.25$$

$$y_1 = y_0 + \underbrace{f(t_0, y_0)}_{\text{slope at prev. point}}(t_1 - t_0) = 1 + [2(1) - 3(0)](0.25) = 1.5$$

$$t_2 = t_1 + h = 0.25 + 0.25 = 0.5$$

$$y_2 = y_1 + f(t_1, y_1)(t_2 - t_1) = \cdots = 2.0625$$

accuracy? we can actually solve $y' = 2y - 3t$ $y(0) = 1$

$$y = \frac{3}{4}(2t+1) + \frac{1}{4}e^{2t}$$

true $y(0.5) = 2.1796$

to improve, use smaller h (more steps)

if $h = 0.01$ (50 steps)

$y(0.5) = 2.1729$